

Syllabus for the Written Test

Sequences and Series of Real Numbers: Sequences and series of real numbers, Convergent and divergent sequences, bounded and monotone sequences, Convergence criteria for sequences of real numbers, Cauchy sequences, absolute and conditional convergence; Tests of convergence for series of positive terms – comparison test, ratio test, root test.

Functions of One Variable: Limit, continuity, differentiation, Rolle's Theorem, Mean value theorem. Taylor's theorem. Maxima and minima.

Real Analysis: Interior points, limit points, open sets, closed sets, bounded sets; completeness of \mathbb{R} , Taylor's and Maclaurin's, domain of convergence.

Functions of Two Real Variables: Limit, continuity, partial derivatives, differentiability, maxima and minima. Homogeneous functions including Euler's theorem.

Vector Calculus: Scalar and vector fields, gradient, divergence, curl and Laplacian. line integrals, surface integrals, Green's, Stokes and Gauss theorems.

Integral Calculus: Integration as the inverse process of differentiation, definite integrals and their properties, Fundamental theorem of integral calculus. Double and triple integrals. Calculating surface areas and volumes using double integrals and applications. Calculating volumes using triple integrals and applications.

Differential Equations: Ordinary differential equations of the first order of the form $y'=f(x,y)$. Bernoulli's equation, exact differential equations, integrating factor, Orthogonal trajectories, Homogeneous differential equations-separable solutions, Linear differential equations of second and higher order with constant coefficients. Cauchy-Euler equation.

Group Theory: Groups, subgroups, Abelian groups, non-abelian groups, cyclic groups, permutation groups; Normal subgroups.

Linear Algebra: Vector spaces, Linear dependence of vectors, basis, dimension, linear transformations, matrix representation with respect to an ordered basis, Range space and null space, rank-nullity theorem; Rank and inverse of a matrix, determinant, solutions of systems of linear equations, consistency conditions. Eigenvalues and eigenvectors. Cayley-Hamilton theorem.