Energy Harvesting through Piezoelectric Material

Rajeev Kumar
School of Engineering
IIT Mandi
Outlines of the presentation

- Introduction
- Energy Scavenging through Vibration
- Thermodynamic of piezoelectric material
- Piezoelectric Frequency Response
- Piezoelectric Energy Harvester
- Engineering Design Process
- Finite Element Analysis of Layered Piezoelectric Material
- Optimization of Piezoelectric Energy Harvester by Genetic Algorithm
Energy harvesting

Energy harvesting or the process of acquiring energy from the surrounding environment has been a continuous human endeavor throughout history.
Introduction (Cond..)

Need of Energy Harvesting

• Growing need for renewable sources of energy
• Proposes several potentially inexpensive and highly effective solutions
• Reduce dependency on battery power
• Complexity of wiring
• Increased costs of wiring
• Reduced costs of embedded intelligence
• Increasing popularity of wireless networks
• Limitations of batteries
• Reduce environmental impact
## Available energy sources in the environment

<table>
<thead>
<tr>
<th>Energy sources</th>
<th>Example</th>
<th>Energy level</th>
<th>Conversion mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient radiation</td>
<td>RF signal</td>
<td>&lt;1µW/cm²</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>Ambient light</td>
<td>Sunlight, Illumination</td>
<td>100mW/cm² (bright sunlight) 100µW/cm² (office illumination)</td>
<td>Photovoltaic</td>
</tr>
<tr>
<td>Vibration</td>
<td>Machine vibration, Human motion</td>
<td>4-800 µW/cm³</td>
<td>Piezoelectric, Electromagnetic, Electrostatic</td>
</tr>
<tr>
<td>Fluid flow</td>
<td>Wind, Ventilation, Piping, Current, Wave</td>
<td>Air: 200-800 µW/cm³</td>
<td>Turbine (electromagnetic), Piezoelectric</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water: 500mW/cm³</td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>Temperature differential</td>
<td>60µW/cm² at 5 °C difference</td>
<td>Thermoelectric, Thermionic, Thermotunnelling</td>
</tr>
<tr>
<td>Pressure variation</td>
<td>Daily atmosphere pressure change</td>
<td>&lt;10 µW/cm³</td>
<td>Unclear</td>
</tr>
</tbody>
</table>
Energy Scavenging through Vibration

- Electro-magnetic
- Electrostatic
- Piezoelectric
### Energy Scavenging through Vibration (Cond..)

<table>
<thead>
<tr>
<th>Type</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric</td>
<td>1. No separate voltage source</td>
<td>1. Micro fabrication processes are not compatible with standard processes and piezo thin films have poor coupling.</td>
</tr>
<tr>
<td></td>
<td>2. Voltages of 2 to 10 Volts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. No mechanical stops</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Highest energy density</td>
<td></td>
</tr>
<tr>
<td>Electrostatic</td>
<td>1. Easier to integrate with electronics and microsystems</td>
<td>1. Separate voltage source required</td>
</tr>
<tr>
<td></td>
<td>2. Voltages of 2 to 10 Volts</td>
<td>2. Mechanical stops needed</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>1. No separate voltage source</td>
<td>1. Max. voltage of 0.1 volt</td>
</tr>
<tr>
<td></td>
<td>2. No mechanical stops</td>
<td>2. Difficult to integrate with electronics and microsystems</td>
</tr>
</tbody>
</table>

From this comparison it is clear that the most desirable conversion method results that piezoelectric one which presents the major number of advantages. So, it is for these reasons that this is currently the best choice to realize the micro vibration driven generator for energy harvesting to power sensor nodes.
### Examples of common vibration sources

<table>
<thead>
<tr>
<th>Vibration source</th>
<th>Peak acceleration (m/s²)</th>
<th>Frequency of peak acceleration (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base of a 5 HP 3-axis machine tool</td>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>Clothes dryer</td>
<td>3.5</td>
<td>120</td>
</tr>
<tr>
<td>Second story of a wood frame office building</td>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>Car engine compartment</td>
<td>12</td>
<td>200</td>
</tr>
<tr>
<td>Blender casing</td>
<td>6.4</td>
<td>121</td>
</tr>
<tr>
<td>Car instrument panel</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Door frame just after door closes</td>
<td>3</td>
<td>125</td>
</tr>
<tr>
<td>Small microwave oven</td>
<td>2.5</td>
<td>121</td>
</tr>
<tr>
<td>HVAC vents in office building</td>
<td>0.2-1.5</td>
<td>60</td>
</tr>
<tr>
<td>Windows next to a busy road</td>
<td>0.7</td>
<td>100</td>
</tr>
</tbody>
</table>
Thermodynamic of piezoelectric material
Direct Piezo Effect:
The phenomenon of generation of a voltage under mechanical stress is referred to as the direct piezoelectric effect.
The mechanical strain produced in the crystal under electric voltage is referred as converse piezoelectric effect.
Pyroelectric Effect

The phenomenon of generation of a electric field, when the temperature of the crystal is raised or lowered is referred to as the Pyroelectric effect.
Piezoelectric Material (Material with Piezoproperties)

Naturally occurring crystals:
Berlineite (AlPO4), Cane sugar, Quartz, Rochelle salt, Topaz, Tourmaline Group Minerals, and dry bone (apatite crystals)

Man-made ceramics:
Barium titanate (BaTiO3), Lead titanate (PbTiO3), Lead zirconate titanate (Pb[Zr\textsubscript{x}Ti1-x]O\textsubscript{3} 0<x<1) - More commonly known as PZT, Potassium niobate (KNbO3), Lithium niobate (LiNbO3), Lithium tantalate (LiTaO3), Sodium tungstate (NaxWO3), Ba\textsubscript{2}NaNb\textsubscript{5}O\textsubscript{5}, Pb\textsubscript{2}KNb5O\textsubscript{15}

Polymer:
Polyvinylidene fluoride (PVDF)
Polarization of Piezoelectric Material

(a) random orientation of polar domains prior to polarization

(b) polarization in DC electric field

(c) remanent polarization after electric field removed

• Converse Piezoelectric Effect
• Poling Process
• Direct Piezoelectric Effect
Applications of Energy Harvesting through Piezoelectric Material

- The best-known application is the electric CIGARETTE LIGHTER: pressing the button causes a spring-loaded hammer to hit a piezoelectric crystal, producing a sufficiently high voltage electric current that flows across a small spark-gap, thus heating and igniting the gas.
- Gas burners now have built-in piezo-based ignition systems.
- Battery-less wireless doorbell push button
- The armed forces toyed with the idea of putting piezoelectric materials in soldiers boots to power radios and other portable electronic gear
• Several nightclubs, mostly in Europe, have already begun to power their strobes and stereos using the force of hundreds of people pounding on piezoelectric lined dance floors.
Applications of Energy Harvesting through Piezoelectric Material (Cond..)

- Several gyms, notable in Portland and a few other places are powered by a combination of piezoelectric set ups and generators set up on stationary bikes.
• Laying piezoelectric crystal arrays underneath sidewalks, stairwells, and pretty much any other high traffic area to power street lights.
Applications of Energy Harvesting through Piezoelectric Material (Cond.)

- Piezoelectric Powered Music Instruments
Applications of Energy Harvesting through Piezoelectric Material (Cond..)

- Capitalizing on the friction and heat created by walking, running and even just wearing jeans, engineers from Michigan Technological University, Arizona State University devised a way to use this type of generated energy to charge portable electronic devices, like iPods and mobile phones.

  - Biomechanical Energy Harvester
  - Energy harvesting by Piezoelectric windmills

Computational Intelligence Applications to Renewable Energy-2012
Engineering Design Process

1. Identify the need or problem
2. Research the need or problem
3. Develop possible solutions
4. Select the best solutions
5. Construct a prototype
6. Test and evaluate the prototype
7. Modify to improve the design if needed
8. Detailed Design
Engineering Design Process (Contd..)

- Analyze the problem
- Optimization of the problem
- Detailed Design
- Detailed Drawing of the problem
Piezoelectric Energy Harvester Model

The electromechanical model of this structure can be represented by the following set of differential equations

\[
\begin{align*}
[m_{uu}]\ddot{q} + [c_{uu}]\dot{q} + \left(k_{uu} + [k_{u\phi_s}]^{-1}k_{\phi_s u}\right)\{q\} &= \{F_m\} + \frac{1}{2}[k_{u\theta}]\theta \\
\{\phi_s\} &= [k_{\phi_s\phi}]^{-1}\left(\frac{1}{2}[k_{\phi_s\theta}]\{\theta\} + [k_{\phi_s u}]\{q\}\right) - \frac{1}{2}[k_{u\phi_s}][k_{\phi_s\phi}]^{-1}[k_{\theta\theta}]\theta - [k_{u\phi_a}]\phi_a \\
\text{Harvested power} \quad P &= \frac{R(\omega Q)^2}{2} \quad Q = f\{\phi_s\}
\end{align*}
\]
Piezoelectric Frequency Response

Piezoelectric energy harvester is only effective under a narrow bandwidth of excitation frequency. If the excitation frequency shifts from this band, the power density of the harvester will significantly decrease.
If the excitation frequency shifts from this band, the power density of the harvester will significantly decrease.

Different strategies have been used to enhance the harvesting performances when the vibration source has a larger frequency bandwidth.

One of them is to use an array of harvesters which consists of multiple harvesters having different resonance frequencies in order to increase the harvested power on a wider frequency bandwidth. However, this leads to a harvester having a higher volume, which decreases its power density (mW.cm-3).

Therefore Design optimization is required.
Piezoelectric Energy Harvester Model (Cond.)

\[ y \]

\[ L_1 \]

\[ L_2 \]

\[ L_p \]

\[ t_p \]

\[ t_1 \]

\[ t_2 \]

\[ x \]

\[ z \]

\[ b_1 \]

\[ b_p \]

\[ b_2 \]
Objective function

- The cost function of this first optimization problem is to maximize the mean power density over a certain frequency bandwidth.
- The power density is defined as the ratio of the harvested Power and the harvester volume.

\[ \gamma_m = \oint_{f_1}^{f_2} \frac{\gamma}{f_2 - f_1} df \quad V = b_1 L_1 t_1 + b_2 L_2 t_2 + 2b_p L_p t_p \]

\[ \gamma_m = \oint_{f_1}^{f_2} \frac{R(\omega Q)^2}{2(f_2 - f_1)(b_1 L_1 t_1 + b_2 L_2 t_2 + 2b_p L_p t_p)} df \]
Finite Element Modeling

Constitutive relation

Converse piezoelectric effect

\[
\{\sigma\}_k = \left[ Q \right]_k \{\varepsilon\}_k - \left[ e \right]_k \{E\}_k - \left[ \lambda \right]_k \Theta
\]

Direct Piezoelectric effect

\[
\{D\}_k = \left[ e \right]_k^T \{\varepsilon\}_k + \left[ b \right]_k \{E\}_k + \left[ P \right]_k \Theta
\]

For a non piezoelectric layer

\[
\{e\}_k = \{0\} \quad \{P\}_k = \{0\} \quad \{E\}_k = \{0\}
\]
Where

$$\bar{e}_k = [T_v]^T [e]_k [T_o]_k [T_\varepsilon] [T_\theta]$$

$$\bar{b}_k = [T_v]^T [b]_k [T_v]$$

$$\{\bar{p}\}_k = [T_v]^T \{p\}_k$$

$$\bar{Q}_k = [T_\theta]^T [T_\varepsilon]^T [T_o]^T \{Q\}_k [T_o]_k [T_\varepsilon] [T_\theta]$$

$$\{\bar{\lambda}\}_k = [T_\theta]^T [T_\varepsilon]^T [T_o]^T \{\lambda\}_k$$

$$[T_\varepsilon]$$ Strain transformation matrix

$$[T_o]$$ Ply orientation transformation matrix

$$[T_\theta]$$ Rotational transformational matrix

$$[T_v]$$ Vector transformation matrix
Finite Element Modeling (Cond..)

Elastic stiffness coefficients matrix

\[
[Q] = \begin{bmatrix}
\frac{E_1}{(1-v_{12}v_{21})} & \frac{v_{12}E_2}{(1-v_{12}v_{21})} & 0 & 0 & 0 & 0 \\
\frac{v_{12}E_2}{(1-v_{12}v_{21})} & \frac{E_2}{(1-v_{12}v_{21})} & 0 & 0 & 0 & 0 \\
0 & 0 & G_{12} & 0 & 0 & 0 \\
0 & 0 & 0 & SFG_{23} & 0 & 0 \\
0 & 0 & 0 & 0 & SFG_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

SF = shear correction factor = \( \frac{5}{6} \)

Piezoelectric coefficient matrix

\[
\{e\} = \begin{bmatrix}
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\
e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\
e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36}
\end{bmatrix}
\]

Stress coefficients vector

\[
\{\lambda\} = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 \\
\lambda_{12} \\
0 \\
0
\end{bmatrix}
\]

Pyroelectric coefficients vector

\[
\{p\} = \begin{bmatrix}
p_1 \\
p_2 \\
p_3
\end{bmatrix}
\]

Stress vector

\[
\{\sigma\} = \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{12} \\
\tau_{23} \\
\tau_{13}
\end{bmatrix}
\]

Dielectric constant matrix

\[
[b] = \begin{bmatrix}
b_1 & 0 & 0 \\
0 & b_2 & 0 \\
0 & 0 & b_3
\end{bmatrix}
\]
Transformation matrices

Strain transformation matrix

\[
[T_e] = \begin{bmatrix}
  l_i^2 & m_i^2 & n_i^2 & l_i m_1 & m_i n_1 & n_i l_1 \\
  l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\
  l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\
 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & (l_1 m_2 + l_2 m_1) & (m_1 n_2 + m_2 n_1) & (n_1 l_2 + n_2 l_1) \\
 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & (l_2 m_3 + l_3 m_2) & (m_2 n_3 + m_3 n_2) & (n_2 l_3 + n_3 l_2) \\
 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & (l_3 m_1 + l_1 m_3) & (m_3 n_1 + m_1 n_3) & (n_3 l_1 + n_1 l_3)
\end{bmatrix}
\]

Rotational transformation matrix

\[
[T_\theta] = \begin{bmatrix}
  t_\theta & 0 & 0 & 0 \\
  0 & t_\theta & 0 & 0 \\
  0 & 0 & t_\theta & 0 \\
 0 & 0 & 0 & [t_\theta]
\end{bmatrix}
\]

Ply orientation transformation matrix

\[
[T_\alpha] = \begin{bmatrix}
  c^2 & s^2 & 0 & cs & 0 & 0 \\
  s^2 & c^2 & 0 & -cs & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  -2cs & 2cs & 0 & c^2 - s^2 & 0 & 0 \\
  0 & 0 & 0 & c & s & 0 \\
  0 & 0 & 0 & 0 & -s & c
\end{bmatrix}
\]

Vector transformation matrix

\[
[T_v] = \begin{bmatrix}
l_1 & m_1 & n_1 \\
l_2 & m_2 & n_2 \\
l_3 & m_3 & n_3
\end{bmatrix}
\]

\[
l, m, n \quad \text{Direction cosines between local and global axis}
\]

\[
c = \cos \theta \quad s = \sin \theta
\]
9-node degenerate shell element

Three Co-ordinate system \( x, y, z \quad x', y', z' \quad \xi, \eta, \zeta \)

Shape Function

\[
N_i = \frac{1}{4}(1 + \xi \xi_i)(1 + \eta \eta_i)(\xi \xi_i + \eta \eta_i - 1) \quad i=1,2,3,4
\]

\[
N_i = \frac{1}{2}(1 - \xi^2)(1 + \eta \eta_i) \quad i=5,7
\]

\[
N_i = \frac{1}{2}(1 - \eta^2)(1 + \xi \xi_i) \quad i=6,8
\]

\[
N_i = (1 - \xi^2)(1 - \eta^2) \quad i=9
\]
Finite Element Modeling (Cond.)

Nodal Coordinates

\[
\begin{align*}
  \{x_i\} & = \frac{1}{2} \left( \begin{array}{c}
    \{x_i\} \\
    \{y_i\} \\
    \{z_i\}_\text{middle} \\
  \end{array} \right) + \left( \begin{array}{c}
    \{x_i\} \\
    \{y_i\} \\
    \{z_i\}_\text{top} \\
  \end{array} \right) \\
  \{x_i\} & = \frac{1}{2} \left( \begin{array}{c}
    \{x_i\} \\
    \{y_i\} \\
    \{z_i\}_\text{bottom} \\
  \end{array} \right)
\end{align*}
\]

\[
\vec{V}_{3i} = \begin{bmatrix} l_{3i} \\ m_{3i} \\ n_{3i} \end{bmatrix} = \frac{1}{t_i} \left( \begin{array}{c}
    \{x_i\} \\
    \{y_i\} \\
    \{z_i\}_\text{top} \\
  \end{array} \right) - \left( \begin{array}{c}
    \{x_i\} \\
    \{y_i\} \\
    \{z_i\}_\text{bottom} \\
  \end{array} \right)
\]

\[
t_i = \left( (x_{i\text{top}} - x_{i\text{bottom}})^2 + (y_{i\text{top}} - y_{i\text{bottom}})^2 + (z_{i\text{top}} - z_{i\text{bottom}})^2 \right)^{1/2}
\]

9-node degenerate shell element
Relation between the Co-ordinate systems

\[
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix} = \sum_{i} N_i \begin{bmatrix}
  x_i \\
  y_i \\
  z_i \\
\end{bmatrix} + \sum_{i} N_i \frac{\zeta}{2} t_i \vec{V}_{3i}
\]

\[
\vec{V}_3 = \begin{bmatrix}
  l_3 \\
  m_3 \\
  n_3 \\
\end{bmatrix} = \frac{\sum_{i=1}^{9} N_i \vec{V}_{3i}}{\sum_{i=1}^{9} N_i \vec{V}_{3i}} \\
\vec{V}_1 = \begin{bmatrix}
  l_1 \\
  m_1 \\
  n_1 \\
\end{bmatrix} = \frac{\vec{i} \times \vec{V}_3}{|\vec{i} \times \vec{V}_3|} \\
\vec{V}_2 = \begin{bmatrix}
  l_2 \\
  m_2 \\
  n_2 \\
\end{bmatrix} = \vec{V}_3 \times \vec{V}_1
\]
Displacement Field

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} = \sum_{i}^{nnel} N_{i} \begin{bmatrix}
  x_{i} \\
  y_{i} \\
  z_{i}
\end{bmatrix}_{np} + \sum_{i} N_{i} H_{i} \vec{V}_{3i}
\]

\[H_{i} = t_{ok_{i}} + \frac{\zeta_{k}}{2} t_{k_{i}}\]

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} = \sum_{i=1}^{nnel} N_{i} (\xi, \eta) \begin{bmatrix}
  u_{i} \\
  v_{i} \\
  w_{i}
\end{bmatrix} + H_{i} \zeta \begin{bmatrix}
  \vec{V}_{1i} \\
  -\vec{V}_{2i}
\end{bmatrix} \begin{bmatrix}
  \alpha_{i} \\
  \beta_{i}
\end{bmatrix}
\]

\(\zeta = 1\)

\(\zeta_{k} = 1\)

\(\zeta_{k} = 0\)

\(\zeta_{k} = -1\)

\(\zeta = 0\)

\(\zeta = -1\)
**Finite Element Modeling (Cond..)**

**Displacement & Strain Field**

\[
\{u\}_e = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_i^{\text{nnel}} \begin{bmatrix} N_i & 0 & 0 \\ 0 & N_i & 0 \\ 0 & 0 & N_i \end{bmatrix} \begin{bmatrix} -l_{2i}N_iH \\ -m_{2i}N_iH \\ -n_{2i}N_iH \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = [N_{ul}]_e \{q\}_e
\]

\[
\{\varepsilon\} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} + \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} \end{bmatrix}
\]
Finite Element Modeling (Cond..)

Strain Field

$$\{\varepsilon\} = \sum_{i=1}^{n_{nel}} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & -l_{2i} g_{xi} & l_{li} g_{xi} \\ 0 & \frac{\partial N_i}{\partial y} & 0 & -m_{2i} g_{yi} & m_{li} g_{yi} \\ 0 & 0 & \frac{\partial N_i}{\partial z} & -n_{2i} g_{zi} & n_{li} g_{zi} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & -(l_{2i} g_{yi} + m_{2i} g_{xi}) & (l_{li} g_{yi} + m_{li} g_{xi}) \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} & -(m_{2i} g_{zi} + n_{2i} g_{yi}) & (m_{li} g_{zi} + n_{li} g_{yi}) \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} & -(n_{2i} g_{xi} + l_{2i} g_{zi}) & (n_{li} g_{xi} + l_{li} g_{zi}) \end{bmatrix} \begin{bmatrix} u_{oi} \\ v_{oi} \\ w_{oi} \\ \beta_i \\ \alpha_i \end{bmatrix} = [B]_e \{q\}_e$$
Temperature distribution is taken linear within the element.

Using the shape function the temperature of any point in the element can be uniquely given in terms of nodal temperature and gradient of the mid plane as

\[
\Theta = \sum_{i}^{nnel} N_i \Theta_i + \sum_{i}^{nnel} N_i H \varphi_i
\]

\[
\Theta_i \quad \varphi_i \quad \text{are the mid plane temperature and gradient respectively at node } i
\]

\[
\Theta = \sum_{i}^{nnel} \begin{bmatrix} N_i & 0 \end{bmatrix} \begin{bmatrix} \Theta_i \\ \varphi_i \end{bmatrix} + \sum_{i}^{nnel} \begin{bmatrix} 0 \\ N_i H \end{bmatrix} \begin{bmatrix} \Theta_i \\ \varphi_i \end{bmatrix} = [N_\Theta]_e \{\theta\}_e + [N_\varphi]_e \{\theta\}_e
\]
Electric Field

Electric field in the $k^{th}$ piezoelectric layer within the element can be given as

$$\{E\}_k = -\{B_\phi\}_e \phi_{p_k}$$

Where

$$\{B_\phi\}_e = \frac{1}{t_{p_k}} \begin{pmatrix} l_3 \\ m_3 \\ n_3 \end{pmatrix}$$

- $t_{p_k}$ Thickness of $k^{th}$ piezoelectric layer
- $\phi_{p_k}$ Electric potential of $k^{th}$ piezoelectric layer
Electric Field

Electric field in the $k^{th}$ piezoelectric layer within the element can be given as

$$\{E\}_k = -\{B_\phi\}_e \phi_{p_k}$$

Where

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- $t_{p_k}$ Thickness of $k^{th}$ piezoelectric layer
- $\phi_{p_k}$ Electric potential of $k^{th}$ piezoelectric layer
Strain energy

Using the variational principle the potential energy is given as

\[ V = \frac{1}{2} \int \sum_k \int \{\varepsilon\}^T \{\sigma\}_k d\zeta dA \]

Substituting various values

\[ V = \frac{1}{2} \left[ \{q\}_e^T [k_{uu}]_e \{q\}_e + \{q\}_e^T [k_{u\phi}]_e \{\phi\}_e - \{q\}_e^T [k_{u\theta}]_e \{\theta\}_e \right] \]

where

\[ [k_{uu}]_e = \sum_{k=1}^{nl} \int [B]^e [Q]_k [B]_e dV \]
Strain energy

\[
[k_{u\theta}]_e = \sum_{k=1}^{n_l} \int_V \left( [B]^T_e \{\bar{\lambda}\}_k [N_\theta]_e + H[B]^T_e \{\bar{\lambda}\}_k [N_\phi]_e \right) dV
\]

\[
[k_{u\phi}]_e = \begin{bmatrix}
\int [B]^T_e [\bar{\epsilon}]_{p_1}^T \{B_\phi\}_e dV \\
\int [B]^T_e [\bar{\epsilon}]_{p_2}^T \{B_\phi\}_e dV \\
\ldots \\
\int [B]^T_e [\bar{\epsilon}]_{p_{npl}}^T \{B_\phi\}_e dV
\end{bmatrix}
\]

\[
\{\phi\}_e = \begin{bmatrix}
\phi_{p_1} \\
\phi_{p_2} \\
\ldots \\
\phi_{p_{npl}}
\end{bmatrix}
\]
Electrical energy

The element electrical energy can be given as

\[
W^e = \frac{1}{2} \int \sum_{k}^{t_k} \int_{t_{k-1}} A \{E\}_e^T \{D\}_e d\Gamma dA
\]

Substituting various values

\[
W^e = -\frac{1}{2} \{\phi\}_e^T [k_{\phi u}]_e \{q\}_e + \frac{1}{2} \{\phi\}_e^T [k_{\phi \phi}]_e \{\phi\}_e - \frac{1}{2} \{\phi\}_e^T [k_{\phi \theta}]_e \{\theta\}_e
\]

where

\[
[k_{\phi u}]_e = [k_{u \phi}]_e^T
\]
Finite Element Modeling (Cond..)

Electrical energy

\[
[k_{\phi\phi}]_e = \begin{bmatrix}
\int_{V} \{B_\phi\}^T_e \bar{b} p_1 \{B_\phi\}_e dV \\
\int_{V} \{B_\phi\}^T_e \bar{b} p_2 \{B_\phi\}_e dV \\
\vdots \\
\int_{V} \{B_\phi\}^T_e \bar{b} p_{npl} \{B_\phi\}_e dV
\end{bmatrix}
\]
Finite Element Modeling (Cond.)

Electrical energy

\[
\begin{bmatrix}
\int_V \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_1} \left[ N_\Theta \right]_e + H \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_1} \left[ N_\phi \right]_e \right) dV \\
\int_V \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_2} \left[ N_\Theta \right]_e + H \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_2} \left[ N_\phi \right]_e \right) dV \\
\int_V \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_{npl}} \left[ N_\Theta \right]_e + H \left( \{B_\phi\}_e^T \{\vec{p}\} \right)_{p_{npl}} \left[ N_\phi \right]_e \right) dV
\end{bmatrix}
\]

\[
\left[ k_{\phi\theta} \right]_e =
\]
Kinetic energy

The element kinetic energy can be given as

\[ T = \frac{1}{2} \sum_{k=1}^{n_l} \int_V (\rho_k (\ddot{u}^2 + \ddot{v}^2 + \ddot{w}^2)) dV \]

Substituting various values

\[ T = \frac{1}{2} \{q\}_e^T \begin{bmatrix} m_{uu} \end{bmatrix}_e \{q\}_e \]

\[ [m_{uu}]_e = \sum_{k=1}^{n_l} \int_V \rho_k [N_{\bar{u}e}]^T [N_{\bar{u}e}] \, dV \]

where

\[ \rho_k \quad \text{density of kth layer} \]
Kinetic energy

\[
[N_u^-]_e = \sum_{i}^{n_{nel}} \begin{bmatrix}
N_i & 0 & 0 & -l_{1i} N_i H & l_{1i} N_i H \\
0 & N_i & 0 & -m_{2i} N_i H & m_{2i} N_i H \\
0 & 0 & N_i & -n_{2i} N_i H & n_{2i} N_i H \\
0 & 0 & 0 & N_i & n_{1i} N_i H
\end{bmatrix}
\]
Work done

Work done by the external force and electrical charge is given as

$$W^s = \{q\}_e \{F_m\}_e + \{\phi\}_e \{F_q\}_e$$

$$\{F_m\}_e = \int_{s_1} \left[N^-_u\right]^T \{f_s\}_e \, ds + \left[N^-_u\right]^T \{f_p\}_e$$

$$\{F_q\}_e = \int_{s_2} \left[B\phi\right]^T \{f_q\}_e \, ds$$

\{f_s\}_e \quad \text{Element surface force intensity vector}

\{f_p\}_e \quad \text{Element point load vector}

\{f_q\}_e \quad \text{Element surface electrical charge density vector}
Using Hamilton’s principle

\[ \int_{t_o}^{t_f} \delta (L + W^s) \, dt = 0 \]
\[ \int_{t_o}^{t_f} \left( \delta T - \delta V + \delta W^e + \delta W^s \right) \, dt = 0 \]

The governing equation for an element can be written as

\[
\begin{align*}
\left[ m_{uu} \right]_e \{ \ddot{q} \}_e + \left[ k_{uu} \right]_e \{ q \}_e + \left[ k_{u\phi} \right]_e \{ \phi \}_e - \frac{1}{2} \left[ k_{u\theta} \right]_e \{ \theta \}_e &= \{ F_m \}_e \\
\left[ k_{\phi u} \right]_e \{ q \}_e - \left[ k_{\phi\phi} \right]_e \{ \phi \}_e + \frac{1}{2} \left[ k_{\phi\theta} \right]_e \{ \theta \}_e &= \{ F_q \}_e
\end{align*}
\]
Finite Element Modeling (Cond..)

Governing equations

- Sensors and actuators are present
- vector can be partitioned
- No charge accumulates on the sensor layer

\[
\begin{align*}
[m_{uu}]\ddot{\{q\}} + [k_{uu}]\{q\} + [k_{u\phi_s}]\{\phi_s\} &= \{F_s\} + \frac{1}{2} [k_{u\theta}]\{\theta\} - [k_{u\phi_a}]\{\phi_a\} \\
[k_{\phi_s u}]\{q\} - [k_{\phi_s \phi_s}]\{\phi_s\} + \frac{1}{2} [k_{\phi_s \theta}]\{\theta\} &= \{0\} \\
[k_{\phi_a u}]\{q\} - [k_{\phi_a \phi_a}]\{\phi_a\} + \frac{1}{2} [k_{\phi_a \theta}]\{\theta\} &= \{F_{qa}\}
\end{align*}
\]
Governing equations

Sensor equation

\[ \{\phi_s\} = [k_{\phi,\phi}]^{-1}\left(\frac{1}{2}[k_{\phi,\theta}]\{\theta\} + [k_{\phi,u}]\{q\}\right) \]

Actuator equation

\[ [m_{uu}]\ddot{q} + [c_{uu}]\dot{q} + ([k_{uu}] + [k_{u\phi_s}] [k_{\phi,s}]^{-1} [k_{\phi,u}])\{q\} = \{F_m\} + \frac{1}{2} [k_{u\theta}]\{\theta\} \]

\[ -\frac{1}{2} [k_{u\phi_s}] [k_{\phi,s}]^{-1} [k_{\phi,\theta}]\{\theta\} - [k_{u\phi_a}]\{\phi_a\} \]
Flowchart of Genetic Algorithm

START → Parameter representation → Initial population → Evaluate cost

Pairing → Natural selection → Ranking

Mating → Mutation → Evaluate cost → Convergence

END
GA Optimization with MATLAB

- \( [x, fval, exitflag] = ga(@fitnessfun, nvars, A, b, Aeq, beq, LB, UB, nonlcon, options) \)
- \( \text{options} = \	ext{gaoptimset} \)

![MATLAB GUI for GA Optimization](image)

Choose GA solver

Enter problem and constraints

Run solver

View results and final points

Specify options
GA Optimization w/ Matlab—Population

- options=gaoptimset(‘PopulationType’, ‘bitstring’, ‘CreationFcn’, @gacreationlinearfeasible)

- For a nonlinearly constrained problem, ‘bit string’ cannot be used for population.
- For a linearly constrained problem, use feasible population for creation function.
GA Optimization w/ Matlab—Reproduction & Mutation

- `options=gaoptimset('CrossoverFraction', 0.8)`
- `options=gaoptimset('MutationFcn', @mutationuniform)`

<table>
<thead>
<tr>
<th>Options</th>
<th>Crossover</th>
<th>Mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>1100101010</td>
<td>0011011001</td>
</tr>
<tr>
<td>Fitness scaling</td>
<td>1011101110</td>
<td>00110110101</td>
</tr>
<tr>
<td>Selection</td>
<td>1100101110</td>
<td>0011001001</td>
</tr>
<tr>
<td>Reproduction</td>
<td>1011101110</td>
<td>0011001001</td>
</tr>
</tbody>
</table>

Elite count: Use default: 2
Crossover fraction: Use default: 0.8
Mutation function: Uniform
Rate: Adaptive feasible Custom
GA Optimization w/ Matlab—Crossover

- `options = gaoptimset('CrossoverFcn', @crossoversinglepoint)`

![Crossover Options](image)

Crossover function: Single point

- Single point
- Scattered
- Two point
- Intermediate
- Heuristic
- Arithmetic
- Custom

Crossover:

| 1100101010 |
| 1011101110 |

| 1100101110 |
| 1011101110 |
• \texttt{options=gaoptimset(‘SelectionFcn’, @selectionroulette)}
Maximize

\[ \gamma_m = \int_{f_1}^{f_2} \frac{R(\omega Q)^2}{2(f_2 - f_1)(b_1 L_1 t_1 + b_2 L_2 t_2 + 2b_p L_p t_p)} \, df \]

Subject to:

- \( 10 \leq L_2 \leq 40 \)
- \( 0.75 \leq t_1 \leq 2.00 \)
- \( 1 \leq R \leq 10 \)

\( f_1 \) and \( f_2 \) are arbitrary set to 90 and 110 Hz.
## Material Properties

<table>
<thead>
<tr>
<th>Harvester properties</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass density</td>
<td>$\rho_b$</td>
<td>8410 kg.m$^{-3}$</td>
</tr>
<tr>
<td>PZT-5H density</td>
<td>$\rho_p$</td>
<td>7800 kg.m$^{-3}$</td>
</tr>
<tr>
<td>Brass modulus of elasticity</td>
<td>$E_b$</td>
<td>103.4 GPa</td>
</tr>
<tr>
<td>PZT-5H modulus of elasticity</td>
<td>$E_p$</td>
<td>62 GPa</td>
</tr>
<tr>
<td>Permittivity of ceramics</td>
<td>$\epsilon^s$</td>
<td>27.3 nF.m$^{-1}$</td>
</tr>
<tr>
<td>Piezoelectric charge constant</td>
<td>$d_{31}$</td>
<td>-320 pC.N$^{-1}$</td>
</tr>
<tr>
<td>Width of the piezoelectric ceramic</td>
<td>$b_p$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Length of the piezoelectric ceramic</td>
<td>$L_p$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Thickness of the piezoelectric ceramic</td>
<td>$t_p$</td>
<td>0.27 mm</td>
</tr>
<tr>
<td>Width of the first section of the beam</td>
<td>$b_1$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Width of the second section of the beam</td>
<td>$b_2$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Length of the first section of the beam</td>
<td>$L_1$</td>
<td>51 mm</td>
</tr>
<tr>
<td>Thickness of the second section of the beam</td>
<td>$t_2$</td>
<td>5 mm</td>
</tr>
</tbody>
</table>
Simulation parameters for the genetic algorithm optimization

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of individuals in a population</td>
<td>12</td>
</tr>
<tr>
<td>Number of bits by genes</td>
<td>8</td>
</tr>
<tr>
<td>Proportion of mutation</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of generation having the same maximum</td>
<td>150</td>
</tr>
</tbody>
</table>
Results of optimization problem

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Single harvester</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of the first section of the beam</td>
<td>$t_1$</td>
<td>1.21 mm</td>
</tr>
<tr>
<td>Length of the second section of the beam</td>
<td>$L_2$</td>
<td>17.56 mm</td>
</tr>
<tr>
<td>Resistive load</td>
<td>$R$</td>
<td>6.39 kΩ</td>
</tr>
</tbody>
</table>

Harvest a power 75% higher
Conclusion

- From the overview, it is quite clear that piezoelectric energy harvesting has great potential at micro level and some very important part of applications are still in the research and development stage.

- The ability of piezoelectric equipment to convert motion from human body into electrical power is remarkable.

- It is a great hope that energy harvesting will rule the next decade in the technical field.


Thank you