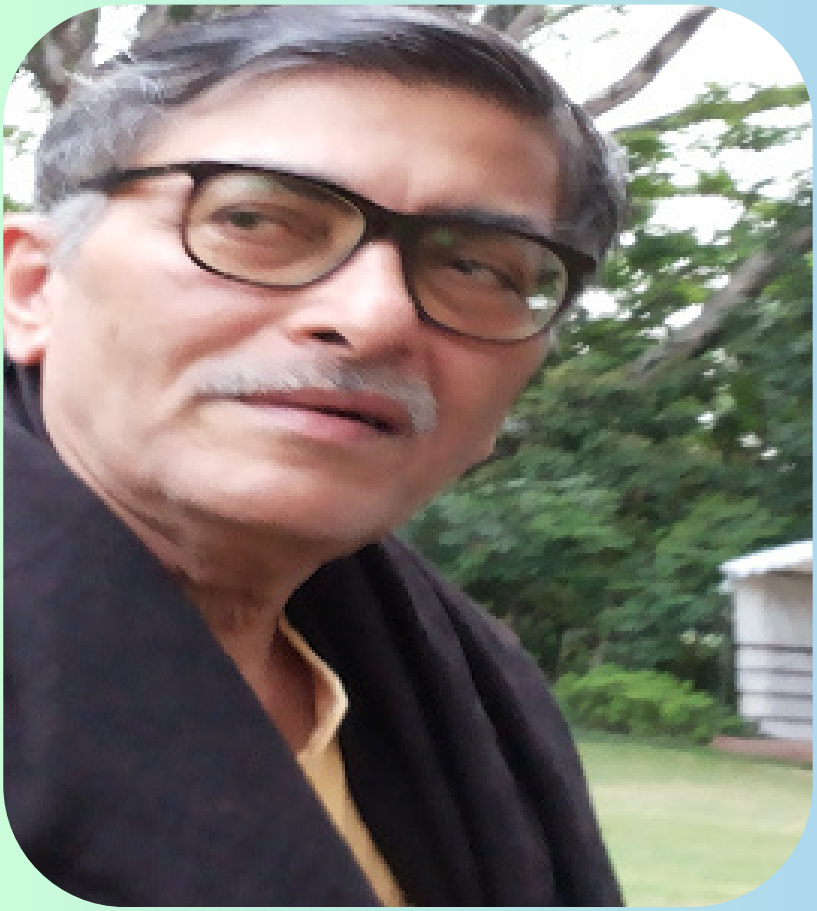
भारतीय प्रौद्योगिकी संस्थान मंडी

INDIAN INSTITUTE OF TECHNOLOGY MANDI

Institute Colloquium

“Teaching calculus as ganita”



18th March, 2025

05:00PM

Auditorium, North Campus

Prof. C. K. Raju

Honorary Professor, Indian Institute of Education, Ex-Tagore Fellow, Indian Institute of Advanced Study

Professor C. K. Raju holds a BSc (Hons) in physics, an MSc in mathematics from Mumbai, followed by a PhD from the Indian Statistical Institute, Kolkata.

He initially taught and researched in formal mathematics (analysis, functional analysis, Schwartz distributions) and its application to general relativity and quantum field theory for several years. Later he joined C-DAC to play a key role in building the first Indian supercomputer Param. He was responsible for porting applications of national importance (space, oil, etc.), and that experience led him to abandon formal mathematics.

In *Time: Towards a Consistent Theory* (Kluwer Academic, Dordrecht, 1994) he pointed out that existing physics must use functional differential equations, with a tilt in the arrow of time, and this nearly explains quantum mechanics. In the *Eleven Pictures of Time* (Sage, 2003) he proposed a new way to relate science and religion through time.

In *Cultural Foundations of Mathematics* (Pearson Longman, 2007) he proposed a new realistic philosophy of math, called zeroism, and compiled evidence for the origin and development of calculus in India and its transmission to Europe. He further explained that the West had immense difficulty in understanding imported Indian calculus and its infinite series, and eventually evolved a (bad) solution to suit its cultural and political convenience, but calculus becomes very easy by reverting to its original understanding. His shorter books include *Is Science Western in Origin?* (Multiversity 2010), *Ending Academic Imperialism* (Citizens International, 2011) and *Euclid and Jesus* (Multiversity, 2012).

His books have been highly praised, and his several articles have also drawn high praise from both referees and readers.

Over the last decade, as part of the Multiversity movement, he has developed and taught several decolonised courses including on calculus, geometry, statistics, and also the history and philosophy of science. Accounts of this can also be found in videos of various lectures, expository and popular level articles, media reports, press coverage, and his blog,

He has long been a professor in various departments, including mathematics, in universities in India and abroad. He headed the largest computer science department in India with over 38000 students. He is an Honorary Professor (Indian Institute of Education), and an Emeritus Professor (SGT University), and was a Tagore Fellow at the Indian Institute of Advanced Study, Shimla. Earlier, too, he was a Fellow at IIAS and on the editorial board of the Journal of Indian Council of Philosophical Research.

He has lectured on six continents, ranging from prominent universities and institutes such as MIT, ICMC Brazil, Cape Town, UNISA, Durban, Auckland, Australian National University, National University of Singapore, University Sains Malaysia, Indian Institute of Science, TIFR, Max Planck Institutes, Munich, Berlin, Dresden, Amsterdam, Lyon, ICTP, Trieste, Tehran, Bethlehem, Beirut, to Soweto, and refugee camps in Palestine, and remote village schools in India. He interacts with non-academics as in his conversations with the Dalai Lama, or with artists at a music festival in Berlin.

He has received numerous honours and awards, including the TGA award in Hungary in 2010, for correcting Einstein's mistake, Fellowship of the Institute of Complex Thought, Lima, Bhartiya Dharohar Award, MP Ratna, etc.

Teaching calculus as ganita

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Extended abstract

Sice my 2000 keynote at Hawaii U, the related article,[1] my 2007 book,[2] the related encyclopedia articles,[3] various talks including at MIT, Cambridge, Mass,[4] I have been pointing out the Indian origin of calculus, and its brazen intellectual theft by Europeans. The further developments/clarifications of last 25 years are summarised in these two talks[5] (related tweets 1, 2) at the 9th Pacific Rim Conference on Math, in Darwin, Australia, in June last year.

I have also **taught** calculus differently,[6] since calculus as it originated in India ≠ calculus as taught today in IITs. The original is BETTER and far EASIER. Teaching the original Indian understanding of calculus as ganita enables students to solve HARDER problems NOT covered in usual college or IIT calculus courses.[7] Simple examples included as tutorials in the calculus as ganita course, are non-elementary elliptic functions needed for the solution of the simplest problem of Newtonian physics, the simple pendulum,[8] or the problem of ballistics with resistance, or brachistochrone with resistance[9] etc. This course has been successfully demonstrated in 5 universities in 3 countries, including two Muslim countries, Malaysia and Iran.

More difficult problems, insoluble with the current calculus (or even the theory of Schwartz distributions[10]), are explained in my book,[11] but not included in the course. One such question is how to understand the nonlinear partial differential equations of physics, such as the Navier Stokes equations, at a shock wave, or discontinuity in real fluids with viscosity and thermal conductivity, which equations cannot be reduced to a quasi-linear "conservation form". I have derived new junction conditions[12] for relativistic shocks earlier using non-standard analysis,[13] later similar conditions with ganita.

To quickly recapitulate, calculus originated in India in the 5th c., with the work of the LOWER caste[14] Aryabhata[15] who used finite differences and a recursive numerical method[16] (today falsely called Euler's method) to solve differential equations, to derive sine values precise to the first sexagesimal minute (about 5 decimal places). Note that numerically solving differential equations is at the heart of all calculus applications today, such as sending a man to the moon, **not** formulae for derivatives and integrals of **elementary** functions as are commonly taught, nor even infinite series.[17]

No one used the term Indian "calculus" before my 1998 INSA project.[18] Further, **all** other authors on Indian calculus have still missed out the key next step of calculus as ganita (which also separates it from numerical analysis). This key step was by Brahmagupta[19] (7th c.): who introduced अव्यक्त गणित (or polynomial arithmetic). Polynomial arithmetic is naturally non-Archimedean,[20]unlike axiomatic "real" numbers which are the largest Archimedean ordered field. The important point: non-Archimedean arithmetic has infinities and infinitesimals, hence limits in the sense of elementary real analysis[21] are impossible. Therefore, this course was earlier (since 2009)[22] called "Calculus without limits",[23] to point out the fundamental difference from current calculus and analysis courses based on limits. However, very few people have a good enough grasp of real analysis to have understood this in the last 15 years. Infinities and infinitesimals arise also in Robinson's non-standard analysis,[24] which makes calculus easy except for the very first step of learning non-standard analysis which is extraordinarily difficult, too hard[25] even for most graduate students in axiomatic mathematics. In contrast, polynomial arithmetic is middle school stuff. This is to be combined with zeroism,[26] earlier called śūnyavāda.

Before calculus backward Europeans learnt even efficient **primary-school** arithmetic and place-value ("Arabic numerals") from India, beginning with 10th c. Gerbert[27] and the 13th c. Fibonacci.[28] Europeans then were stuck with inefficient arithmetic with "Roman numerals" which early Greeks learnt from their Persian conquerors to pay them tax. It took another 500 years of intellectual struggle ("first math war")[29] between the Graeco-Roman abacus, and Indian "algorismus", for Europeans to assimilate Indian primary-school arithmetic which they really began to grasp only in the 16th century **after** Vasco da Gama's arrival and extensive contact with India, as depicted in Gregor Riesz *Margarita Philosophica.*[30] But some prominent European mathematicians remained bewildered even about arithmetic until the 20th century.

Thus, the algorismus, i.e., al Khwarizmi's limited understanding of Indian arithmetic, lacked negative numbers, hence so did Fibonacci.[31] Hence, also Leonhard Euler[32] blundered about negative numbers foolishly writing that $-1>\infty$. Many others matched his foolishness, including Augustus de Morgan[33] who was foolishly asserting in the 19th c. that $-9<0$ is impossible since "something cannot be less than nothing". He got the East India Company to finance the publication of an extraordinarily foolish book[34] which obtained maxima and minima without using negative numbers! When even such prominent European mathematicians failed to understand basic ordering among integers how could they have understood ordering among polynomials? However, our whole colonial or पादरीवादी education system is founded on the fanatical 19th c. Macaulie that despite evidence of such "immeasurable stupidity", of the West, which failed to properly understand even Indian primary school arithmetic, till the 19h c., the West was nevertheless somehow "immeasurably superior" in mathematics and science!

However, those who go by facts and evidence are bound to ask: if Europeans had so much difficulty in understanding Indian primary school arithmetic, how could they have *understood* the Indian calculus, even after stealing it and claiming its ownership on the vile dogma of "Christian discovery"[35]? Indeed, the West has not understood the Indian calculus to this day. What part of calculus did Newton and Leibniz, its Christian discoverers,[36] not understand? The part about infinities and infinitesimals (or Indian "non-Archimedean" arithmetic): (1) how to sum an infinite series, as Nīlakaṇṭha did[37] and (2) the meaning of Newton's excessively confused "infinitesimal" fluxions. Many people such as Berkeley,[38] Marx,[39] and Dedekind[40] pointed out Newton's conceptual confusion regarding calculus.

Dedekind's invention of real numbers admitted two key points. 1. That the prevailing understanding of calculus in the late 19th century was completely unacceptable even to Europeans struggling with negative numbers. 2. **That there are no axiomatic proofs in the "Euclid" book,**[41] not even in its first proposition.

It is difficult to understand why such a BIG and obvious lie ("the Euclid book gave axiomatic proofs") is essential to the foundations of axiomatic mathematics. Why must this lie be still taught compulsorily to schoolchildren that the "Euclid" book has (or intended) some axiomatic proofs when it has none? None were intended, for the axiomatisation of the book by Hilbert[42] or Birkhoff[43] fundamentally altered it. Obviously, Indian math educators have no conscience whatsoever (or no knowledge whatsoever or both).

Why continue with this lie as in use of axiomatic real numbers? The simple answer is that if the absence of axiomatic proofs in the "Euclid" book is admitted people will naturally look for the actual source of axiomatic proof, which is in the church and its Crusading theology of reason[44] (falsely read into the "Euclid" book to give it a fake Greek ancestry). The church obviously benefited from the key aspect of axiomatic proof which is not the use of reason, or rigour, as advertised, but the **prohibition of the empirical.**[45] The axiomatic method allowed the church padres to control theology, and thereby tell people what their god wanted, so as to make them behave in a way that suited the political interests of the padres. Likewise, the axiomatic method in mathematics allows Western mathematicians to control the **content** of mathematics, and thereby control the more metaphysical aspects of science dependent on that math.[46]

The other question is why axiomatic proof is regarded as "superior" to scientific proof which accepts reason but also the empirical, like Indian ganita. The sole justification for this belief is Christian superstition,[47] as in White supremacy.[48] Axiomatic mathematicians have been shamelessly running away from a debate on this question for the last decade.[49] The half-truth offered is that the empirical is fallible; but a complex deductive task is far **more** fallible[50] as demonstrated by the fact that students so easily flunk in mathematics, and that every human being without fail loses to a computer in deductive chess.

Dodging such a debate on the purported epistemic or practical value of church-origin axiomatic proof is not only a sign of extreme dishonesty of axiomatic mathematicians (most university mathematicians), it is damaging to the national interest. The ease of Indian calculus makes it of practical value to all others. Chinese could use it, they are not scared like Indians of the West. Blacks can do it they are not all so scared of Whites. Key point: knowledge is not our personal property, if we don't use it, others will and will get ahead of us and we will just be left proudly lamenting "we did it first" (but were too frightened and lacked confidence even to **try** it today!).

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