

: EE 580
: Network Systems: Modelling and Analysis
: 3-0-0-3
: MA 512 Linear Algebra or EE 522 Matrix theory for engineers
and EE509 Linear Dynamical Systems or instructor's consent
: 3 <sup>rd</sup> / 4 <sup>th</sup> year B.Tech. (all branches), MS, M.Tech, Ph.D.
: Discipline elective for B.Tech. (EE, CSE), free elective for others
: Any

**Preamble:** This course serves as a first formal introduction to the broad area of networks. It uses concepts from linear algebra (introduced in MA 512/ EE 522) and graph theory to model complex networks. It touches upon some broad classes of network systems enabling students to see these systems as tractable mathematical models. Further, concepts from EE 509 Linear Dynamical Systems will be extended to study linear networks. Stability of linear and nonlinear networks will also be touched upon. This course could serve as a prerequisite for courses based on control/optimization of network systems or design of multi-agent systems. which could be of interest to students/scholars in the field of Electrical/Mechanical Engineering as well as Computer Science and Engineering.

**Objective:** Recent past has seen the rise of network systems that consist of diverse devices that function independently to collectively achieve an overarching goal. For example, next generation Internet, wireless sensor networks, connected vehicular networks, robot swarms, the social network. Today's engineers are required to analyse the performance, scalability and reliability of these networks, in order to achieve efficient and optimal design. This requires one to understand the underlying properties of these systems.

This course introduces the mathematical background required for understanding properties of network systems. The main components of this course are: Institute of

- Concepts in linear algebra and graph theory
- Averaging and compartmental systems
- Stability of networks

Upon completion of this course students will be able to

appreciate some pivotal results from linear algebra by perceiving their implication in i. the context of networks.

Technology

- ii. use graph theory to model complex networks.
- understand important properties of averaging systems (eg. sensor networks, opinion iii. networks) and compartmental systems (eg. epidemiology, drug kinetics).
- iv. analyse stability of networks.
- v. view complex networks as tractable mathematical models, thereby finding their analysis and design feasible.

Modules: Topics to be covered are:



Introduction (2 hours) Examples of networks systems, engineered (sensor & robotic networks) and natural (social & compartmental networks). Need for tools to understand and analyse these systems.

Review of Matrix theory & Graph theory (4 hours) Jordan normal form, stochastic matrices and spectral radius, Perron-Frobenius theory. Graphs and digraphs, weighted digraphs.

Algebraic graph theory (4 hours) Adjacency matrix, graph theoretical characterization of primitive & irreducible matrices. Elements of spectral graph theory.

Averaging systems (10 hours) Discrete-time and continuous-time averaging systems. Consensus in averaging systems. Convergence properties. Laplacian flow. Design of weighted digraphs. Scalability, optimization. Time-varying and randomized averaging algorithms. Examples: sensor networks, opinion dynamics over social networks.

Compartmental systems (8 hours) Positive systems and Metzler matrices. Compartmental matrices. Dynamic properties. Spectral properties. Algebraic and graphical properties. Examples: epidemiology, drug kinetics.

Stability theory for networks (7 hours) Dynamical systems and stability notions, Lyapunov stability criteria, Krasovskii-LaSalle invariance principle. Linear, nonlinear and linearized systems. Negative gradient systems.

Introduction to simulation (3 hours) Motivation - role of simulation in understanding and engineering network systems. Discussion on available tools MATLAB/Simulink, NetSim, NS2. Design of simulation experiments for sensor networks, computations on models for compartmental systems.

Advanced topics (3 hours) Multi-agent systems, Wireless sensor and actuators networks, robot swarms. Problems and trends. Technol

## **Textbooks:**

- 1. Lectures on Network Systems, Francesco Bullo, Edition 1, 2018, CreateSpace.
- 2. Network Science, Albert-Laszlo Barabasi, 2016, Cambridge University Press.

## **Additional References:**

- 1. The structure and dynamics of networks, Mark Newman, Albert-Laszlo Barabasi and Duncan J. Watts. 2006, Princeton university press.
- 2. Networks An Introduction, Mark Newman, 2018, Oxford University Press.
- 3. Fundamentals of complex networks: models, structures and dynamics, Guanrong Chen, Xiaofan Wang, and Xiang Li, 2014, John Wiley & sons.
- 4. Selected publications to be chosen by the instructor.